$X^T A X + X^T B + C = 0$

where $A = (a_{ij})_{3\times3}$ symmetry matrix

$B = (b_1, b_2, b_3)^T$, C constant

$X = (x_1, x_2, x_3)^T$

suppose $D^T A D = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$ D othorgonal matrix

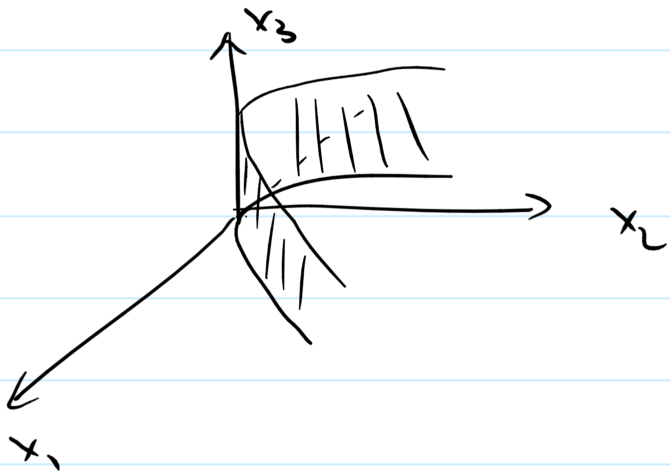
let $X = DY$

$\Rightarrow Y^T \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} Y + Y D^T B + C = 0$

1. $\lambda_2 = \lambda_3 = 0$

$\Rightarrow \lambda_1 x_1^2 + b_1 x_1 + b_2 x_2 + b_3 x_3 + C = 0$

$$\begin{cases} \tilde{x}_1 = x_1 + \dfrac{b_1}{2\lambda_1} \\ -\tilde{x}_2 = b_2 x_2 + b_3 x_3 + \tilde{C} \\ \tilde{x}_3 = x_3 \end{cases} \Rightarrow \lambda_1 \tilde{x}_1^2 = x_2$$

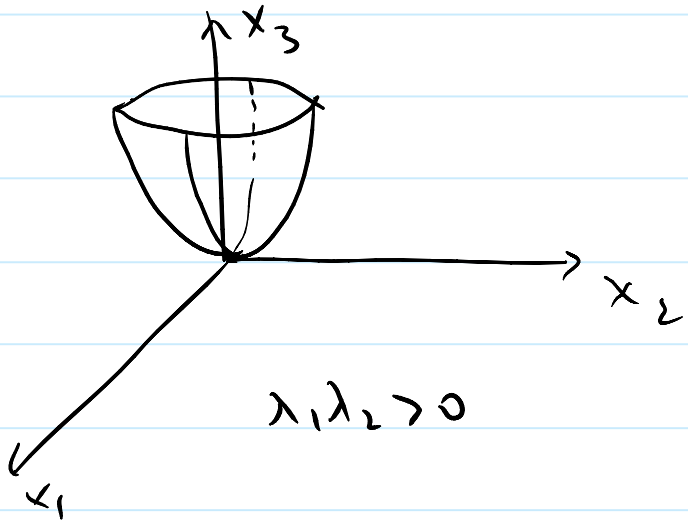


2. $\lambda_3 = 0$

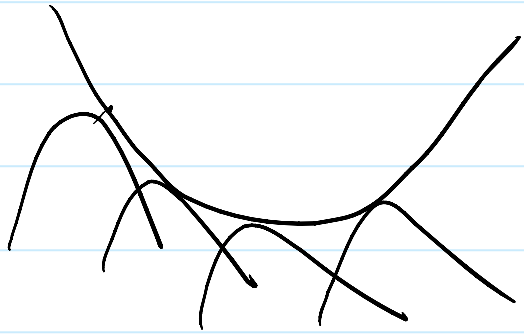
$\lambda_1 x_1^2 + \lambda_2 x_2^2 + b_1 x_1 + b_2 x_2 + b_3 x_3 + C = 0$

$$\begin{cases} \tilde{x}_1 = x_1 + \dfrac{b_1}{2\lambda_1} \\ \tilde{x}_2 = x_2 + \dfrac{b_2}{2\lambda_2} \end{cases} \Rightarrow \tilde{x}_3 = \lambda_1 \tilde{x}_1^2 + \lambda_2 \tilde{x}_2$$

$\left\{ -\tilde{x}_3 = b_3 x_3 + \tilde{c} \right.$



$\lambda_1\lambda_2 > 0$



$\lambda_1\lambda_2 < 0$

3. $\lambda_1\lambda_2\lambda_3 \neq 0$

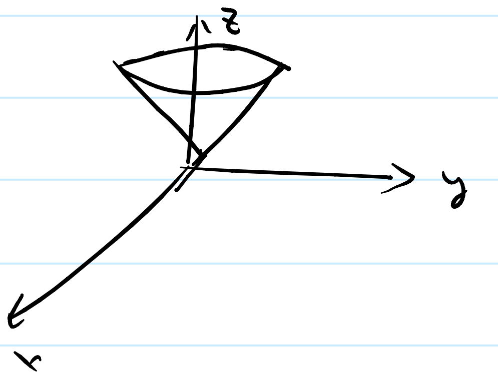
$\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 + b_1 x + b_2 y + b_3 z + c = 0$

$\Rightarrow \lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 = D$ (by translation)

case ①: $D = 0$

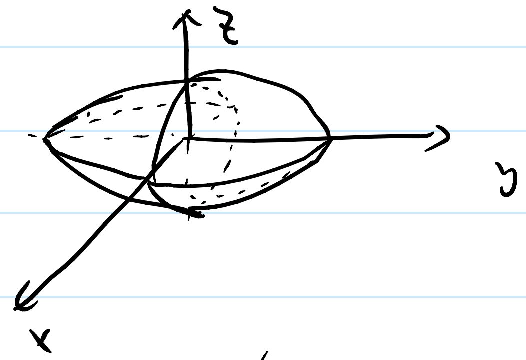
$\lambda_1, \lambda_2, \lambda_3 > 0$ just 1 pt.

$\lambda_1, \lambda_2 > 0$, $\lambda_3 < 0$ $x^2 + y^2 - z^2 = 0$



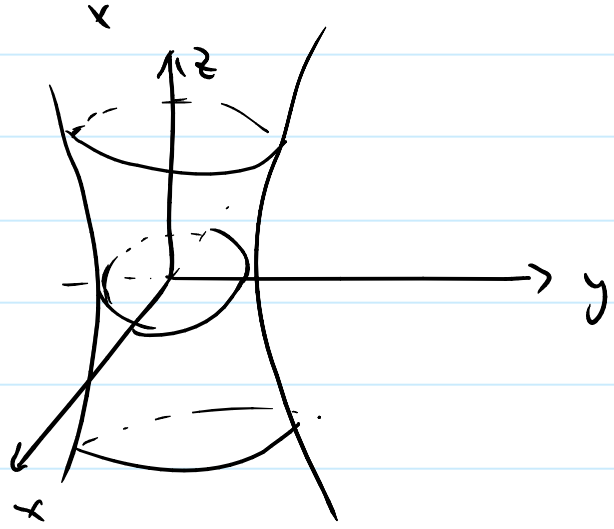
$\lambda_1, \lambda_2, \lambda_3 > 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



$\lambda_1, \lambda_2 > 0 \,,\ \lambda_3 < 0$

$$x^2 + y^2 - z^2 = 1$$



$\lambda_1 > 0 \,,\ \lambda_2, \lambda_3 < 0$

$$-x^2 - y^2 + z^2 = 1$$